TRANSIENT MOTION OF A LIQUID IN A FISSURED POROUS STRATUM SUBJECT TO PERIODIC PRESSURE VARIATION AT THE BOUNDARY

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To determine the physical parameters in a fissured porous stratum, it is necessary to produce periodic pressure variations. This requirement is met by adjusting the rate of flow of liquid into a out of the well. A discussion is given concerning one-dimensional periodic motion of a homogeneous liquid with reference to parameter determination.

1. MOTION IN A RECTILINEAR STRATUM WITH HARMONIC VARIATION AT THE BOUNDARY

A rock of this type may [1] be represented as a continuous medium consisting of a set of joints and a set of blocks, between which there is exchange of liquid. The equations of conservation for the liquid in the two sets are

$$\frac{\partial (m_1\rho)}{\partial t} + \operatorname{div}(\rho \mathbf{V}_1) - q = 0, \qquad \frac{\partial (m_2\rho)}{\partial t} + \operatorname{div}(\rho \mathbf{V}_2) + q = 0. \tag{1.1}$$

Here m_1 and m_2 are the porosities, V_1 and V_2 are the flow speeds, ρ is the density of the liquid, t is time, and q is the mass of liquid emerging into the joints from the blocks in unit time per unit volume of rock. The following expression has been given [1] for q:

$$q = \frac{\rho\alpha}{\mu}(p_2 - p_1). \tag{1.2}$$

Here α characterizes the rate of exchange [1], μ is viscosity, p_1 is the pressure of the liquid in the joints, and p_2 is the pressure in the blocks.

This expression may be put in a different form, without explicit use of the pressure in the porous blocks. Let the pressure in the joints be p; then the relation of q to p is

$$q = \frac{\rho\alpha}{\mu}\left(p_0 - p - \frac{1}{\rho\beta_2}\int_0^t q\, d\theta\right) \tag{1.3}$$

in which β_2 represents the elasticity of the block system and p_0 is the initial pressure in the joints.

The integral term in (1.3) has the meaning of the pressure in the blocks consequent on the influx of liquid. The differential form of (1.3) is

$$\frac{\partial q}{\partial t} + \frac{\alpha}{\mu}\left(\frac{1}{\beta_2} q + \rho\frac{\partial p}{\partial t}\right) = 0. \tag{1.4}$$

Solution of (1.4) subject to $q|_{t=0} = 0$ gives

$$q = -\frac{\rho\alpha}{\mu}\exp\left(-\frac{\alpha}{\mu\beta_2}t\right)\int_0^t \frac{\partial p}{\partial\theta}\exp\left(\frac{\alpha}{\mu\beta_2}\theta\right)d\theta. \tag{1.5}$$

As in [1], we use expressions for V_1 and V_2 in the form of D'Arcy's law and neglect the flow through the blocks [i.e., neglect the second term in the second equation of (1.1)], which gives us from (1.1) and (1.2) an equation describing the motion of a homogeneous and slightly compressible liquid:

$$\frac{\partial p}{\partial t} + \lambda\eta_1\frac{\partial^2 p}{\partial t^2} - \eta_1\frac{\partial}{\partial t}(\nabla^2 p) - \varkappa_1\nabla^2 p = 0, \tag{1.6}$$

$$\lambda = \frac{\mu}{k_1}\beta_1, \quad \eta_1 = \frac{k_1}{\alpha}\frac{\beta_2}{\beta_1+\beta_2}, \quad \varkappa_1 = \frac{k_1}{\mu(\beta_1+\beta_2)},$$

$$\begin{pmatrix}\beta_1 = \beta_{c1} + m_{01}\beta \\ \beta_2 = \beta_{c2} + m_{02}\beta\end{pmatrix}. \tag{1.7}$$

Here k_1 is the permeability of the joint system; β_1, β_{c1}, and β_{c2} are the compressibility coefficients of liquid, joints, and blocks; m_{01} and m_{02} are the initial porosities of joints and blocks; and ∇^2 is the Laplace operator.

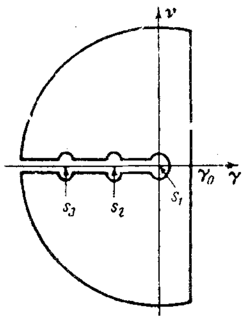

Fig. 1

Consider now the periodic motion in a stratum $0 \le x < \infty$.

The initial pressure is $p = p_0$. The pressure at the boundary x = 0 varies harmonically for t > 0, the amplitude being p_{00} and the circular frequency ω_0:

$$p(0, t) - p_0 = p_{00}\sin\omega_0 t. \tag{1.8}$$

The motion is described by (1.6). The motion of an elastic fluid in an elastic ordinary porous rock subject to (1.8) has already been discussed [2].

We convert to the dimensionless variables

$$\xi = \frac{x}{\sqrt{\varkappa_1 T_0}}, \quad \tau = \frac{t}{T_0}, \qquad P = \frac{p(x,t) - p_0}{p_{00}} \quad \left(T_0 = \frac{2\pi}{\omega_0}\right). \tag{1.9}$$

Equation (1.6) in terms of these becomes as follows for one-dimensional motion in a plane:

$$\frac{\partial P}{\partial\tau} + a\frac{\partial^2 P}{\partial\tau^2} - b\frac{\partial^3 P}{\partial\xi^2\partial\tau} - \frac{\partial^2 P}{\partial\xi^2} = 0, \tag{1.10}$$

$$\left(a = bc, \ b = \frac{\tau_0}{T_0}, \ c = \frac{\delta}{1+\delta}, \ \delta = \frac{\beta_1}{\beta_2}, \right. \qquad (1.10)$$

$$\left. \tau_0 = \frac{\eta_1}{\varkappa_1} = \frac{\eta}{\varkappa}, \ \eta = \frac{k_1}{\alpha}, \ \varkappa = \frac{k_1}{\mu\beta_2} \right). \qquad \text{(cont'd)}$$

Here τ_0 is the characteristic delay time of transient effects in a jointed porous medium [1].

It is stated [1] that η ranges from a few cm^2 up to 10^{10} cm^2, while for $\varkappa \approx 10^4$ cm^2/sec the delay τ_0 ranges from a fraction of a second up to several days.

The initial and boundary conditions are put as follows:

$$P(\xi, 0) = \frac{\partial P(\xi, 0)}{\partial \tau} = P(\infty, \tau) = 0, \qquad (1.11)$$
$$P(0, \tau) = \sin 2\pi\tau .$$

We apply the Laplace transformation to (1.10) and (1.11) to get for $U(\xi, s)$ that

$$\frac{d^2U}{d\xi^2} - \frac{s(1+as)}{1+bs} U = 0,$$

$$\left(U(\xi, s) = \int_0^\infty P(\xi, \tau) e^{-s\tau} d\tau\right). \qquad (1.12)$$

The boundary conditions of (1.11) become

$$U(0, s) = \frac{2\pi}{s^2 + 4\pi^2}, \qquad U(\infty, s) = 0. \qquad (1.13)$$

The solution to (1.12) subject to (1.13) is

$$U(\xi, s) = \frac{2\pi}{s^2 + 4\pi^2} \exp[-\xi Z(s)]$$

$$\left(Z(s) = \left(\frac{s(1+as)}{1+bs}\right)^{1/2}\right). \qquad (1.14)$$

Inversion of (1.14) gives

$$P(\xi, \tau) = \lim_{\nu\to\infty} \frac{1}{i} \int_{\gamma_0 - i\nu}^{\gamma_0 + i\nu} \frac{1}{s^2 + 4\pi^2} \times$$

$$\times \exp\left[-\xi Z(s) + s\tau\right] ds. \qquad (1.15)$$

The integrand of (1.15) in the $s = \gamma + i\nu$ plane has two poles $s = \pm 2\pi i$ and three branch points $s_1 = 0$, $s_2 = -b^{-1}$, $s_3 = -a^{-1}$. The integral of (1.15) is calculated via the contour of Fig. 1. Cauchy's theorem gives

$$P(\xi, \tau) = \sin\left(2\pi\tau - \Delta_1\xi \sin\frac{\psi}{2}\right) \exp\left(-\Delta_1\xi \cos\frac{\psi}{2}\right) +$$

$$+ 2\int_0^{1/b} F(\xi, \tau, \sigma) d\sigma + 2\int_{1/a}^\infty F(\xi, \tau, \sigma) d\sigma. \qquad (1.16)$$

Here

$$F(\xi, \tau, \sigma) = \frac{\exp(-\sigma\tau)}{\sigma^2 + 4\pi^2} \sin\left[\xi\left(\frac{\sigma(1-a\sigma)}{1-b\sigma}\right)^{1/2}\right], \qquad (1.17)$$
$$\Delta_1 = \sqrt[4]{(d_1^2 + d_2^2) d_3^2}$$

$$d_1 = 1 + 4\pi^2 ab, \quad d_2 = 2\pi(b-a), \qquad (1.17)$$
$$d_3 = \frac{2\pi}{1 + 4\pi^2 b^2}, \quad \psi = \operatorname{arc\,tg} \frac{d_1}{d_2}. \qquad \text{(cont'd)}$$

From (1.5) and (1.16) we have for $q(x, t)$ that

$$q = \frac{pk_1 p_\infty}{\mu\varkappa_1 T_0} M(\xi, \tau) \qquad (1.18)$$

in which

$$M(\xi, \tau) = -\frac{1}{b} \exp\left(-\frac{\tau}{b}\right) \int_0^\tau \frac{\partial P(\xi, \theta)}{\partial \theta} \exp\frac{\theta}{b} d\theta =$$

$$= \sum_{j=1}^4 H_j(\xi, \tau),$$

$$H_1(\xi, \tau) = -2\pi(1 + 4\pi^2 b^2)^{-1/2} \sin(2\pi\tau + \operatorname{arc\,ctg} 2\pi b -$$
$$- \Delta_1\xi \sin{}^1/_2\psi) \exp(-\Delta_1\xi \cos{}^1/_2\psi),$$

$$H_2(\xi, \tau) = 2\pi(1 + 4\pi^2 b^2)^{-1/2} \sin(2\pi\tau - \Delta_1\xi \sin{}^1/_2\psi) \times$$
$$\times \exp(-\Delta_1\xi \cos{}^1/_2\psi - \tau/b), \qquad (1.19)$$

$$H_3(\xi, \tau) = 2\int_{1/a}^\infty \frac{\sigma[\exp(-\tau/b) - \exp(-\sigma\tau)]}{(1-b\sigma)(\sigma^2 + 4\pi^2)} \times$$
$$\times \sin[\xi \sqrt{\sigma(1-a\sigma)(1-b\sigma)^{-1}}] d\sigma,$$

$$H_4(\xi, \tau) = 2\int_1^\infty \frac{(\sigma-1)\{\exp(-\tau/b) - \exp[-(\sigma-1)\tau(b\sigma)^{-1}]\}}{(\sigma-1)^2 + 4\pi^2 b^2 \sigma^2} \times$$
$$\times \sin\{\xi \sqrt{(\sigma-1)[c + \sigma(1-c)](b\sigma)^{-1}}\} d\sigma.$$

Formulas (1.16)–(1.19) solve the problem; (1.16) shows that the pressure propagates into the stratum with a phase shift and an amplitude that decreases inwards. The integrals in (1.16) vanish for $t \to \infty$ (nearly steady state) and characterize the effects of the initial conditions.

Consider now approximate formulas for the cases $\tau_0 \ll T_0$ and $\tau_0 \gg\gg T_0$.

The case $\tau_0 \ll T_0$ corresponds to low frequencies, $\omega_0 \ll 2\pi/\tau_0$. We find for the integral terms in (1.16) and (1.18) [3] asymptotic formulas for $\tau \gg 1$ and $\xi \ll \sqrt{\tau}$. We retain the first terms in the expansions, and (with $b \ll 1$) get a solution to (1.16) and (1.19) as

$$P(\xi, \tau) = \sin(2\pi\tau - \xi\sqrt{\pi}) \exp(-\xi\sqrt{\pi}) + 0.25\pi^{-3/2}\xi\tau^{-3/2}, \quad (1.20)$$
$$M(\xi, \tau) = -2\pi\cos(2\pi\tau - \xi\sqrt{\pi}) \exp(-\xi\sqrt{\pi}) - 0.375\pi^{-3/2}\xi\tau^{-5/2}.$$

Similarly, the solution for high frequencies ($\tau_0 \gg T_0$, $\omega_0 \gg 2\pi/\tau_0$, $b \gg 1$) and $\tau/b \gg 1$ becomes

$$P(\xi, \tau) = \sin(2\pi\tau - \xi\Delta_2) \exp(-\xi\Delta_3) + 0.25\pi^{-3/2}\xi\tau^{-3/2}, \quad (1.21)$$
$$M(\xi, \tau) = -b^{-1}\sin(2\pi\tau - \xi\Delta_2) \exp(-\xi\Delta_3) - 0.375\pi^{-3/2}\xi\tau^{-5/2}.$$

Here

$$\Delta_2 = (2b)^{-1/2}[c - 1 + (1 + 4\pi^2 b^2 c^2)^{1/2}]^{1/2},$$
$$\Delta_3 = (2b)^{-1/2}[1 - c + (1 + 4\pi^2 b^2 c^2)^{1/2}]^{1/2}.$$

The solution of (1.20), in accordance with the general conclusion of [1], coincides with the known solution for an ordinary porous medium with a pressure conductivity $\varkappa_1$, because for $\tau_0 \ll T_0$ the delay has a negligible effect on the redistribution of the pressure.

Comparison of (1.20) and (1.21) shows that high-frequency oscillations die out with distance more rapidly than low-frequency ones; but (1.21) shows that this decay in a jointed rock is less rapid than that in an ordinary porous rock characterized by parameter $\varkappa_1$. The reason is that virtually only the liquid in the joints is in motion if $\tau_0 \gg T_0$, and the pressure transmission in the joint system is greater than that in the rock as a whole.

2. ONE-DIMENSIONAL AXIALLY-SYMMETRIC PERIODIC FLOW FROM A BOREHOLE

Consider now the motion of a homogeneous liquid in a horizontally unbounded jointed porous stratum of thickness h penetrated by a vertical borehole of radius r_0.

The initial pressure distribution $p_0(r)$, with r the distance from the axis of the borehole, corresponds to a steady-state flow rate Q_0.

For t > 0 the flow rate at the borehole varies harmonically:

$$Q(r_0, t) = Q_0 + Q_1 \sin \omega_0 t. \tag{2.1}$$

The block permeability is negligible, so the boundary condition for (2.1) is

$$-2\pi \frac{k_1 h}{\mu}\left(r \frac{\partial p}{\partial r}\right)_{r=r_0} = Q_0 + Q_1 \sin \omega_0 t.$$

We convert to the dimensionless variables

$$\zeta = \frac{r}{\sqrt{\varkappa_1 T_0}}, \qquad \tau = \frac{t}{T_0}, \qquad P = \frac{p(r,t) - p_0(r)}{p_{01}}$$

and write (1.6) for this case to get $P(\zeta, t)$ as

$$\frac{\partial P}{\partial \tau} + a \frac{\partial^2 P}{\partial \tau^2} - b\left(\frac{1}{\zeta}\frac{\partial^2 P}{\partial \zeta \partial \tau} + \frac{\partial^3 P}{\partial \zeta^2 \partial \tau}\right) - $$

$$-\left(\frac{1}{\zeta}\frac{\partial P}{\partial \zeta} + \frac{\partial^2 P}{\partial \zeta^2}\right) = 0, \tag{2.2}$$

$$P(\zeta, 0) \quad \frac{\partial P(\zeta, 0)}{\partial \tau} = P(\infty, \tau) = 0,$$

$$\left(\zeta \frac{\partial P}{\partial \zeta}\right)_{\zeta=\zeta_0} = -\sin 2\pi\tau, \tag{2.3}$$

$$\left(p_{01} = \frac{Q_1 \mu}{2\pi k_1 h}, \ \zeta_0 = \frac{r_0}{\sqrt{\varkappa_1 T_0}}\right).$$

The Laplace transformation is again applied to (2.2) and (2.3). The transform

$$U(\zeta, s) = \int_0^\infty P(\zeta, \tau) e^{-s\tau} d\tau$$

satisfies

$$\frac{d^2 U}{d\zeta^2} + \frac{1}{\zeta}\frac{dU}{d\zeta} - \frac{s(1+as)}{1+bs} = 0 \tag{2.4}$$

with the boundary conditions

$$U(\infty, s) = 0, \qquad \left(\zeta \frac{dU}{d\zeta}\right)_{\zeta=\zeta_0} = -\frac{2\pi}{s^2 + 4\pi^2}. \tag{2.5}$$

The solution of (2.4) subject to (2.5), since $K_0'(z) = -K_1(z)$ (in which K_1 and K_0 are MacDonald functions), is

$$U(\zeta, s) = \frac{2\pi}{s^2 + 4\pi^2} \frac{K_0[\zeta Z(s)]}{\zeta_0 Z(s) K_1[\zeta_0 Z(s)]}. \tag{2.6}$$

Inversion gives

$$P(\zeta, \tau) = \lim_{\nu \to \infty} \frac{1}{i} \int_{\gamma_0 - i\nu}^{\gamma_0 + i\nu} \frac{1}{s^2 + 4\pi^2} \frac{K_0[\zeta Z(s)] e^{s\tau} ds}{\zeta_0 Z(s) K_1[\zeta_0 Z(s)]}. \tag{2.7}$$

The contour of Fig. 1 is again used for (2.7); Cauchy's theorem gives

$$P(\zeta, \tau) = \frac{1}{2}\left\{\frac{K_0(\zeta w_1)\exp[i\,{}^1\!/_2\pi(4\tau - 1)]}{\zeta_0 w_1 K_1(\zeta_0 w_1)} - \right.$$

$$\left. - \frac{K_0(\zeta w_2)\exp[-i\,{}^1\!/_2\pi(4\tau + 1)]}{\zeta_0 w_2 K_1(\zeta_0 w_2)}\right\} + \int_0^{1/b} \Phi(\zeta, \tau, \sigma) d\sigma +$$

$$+ \int_{1/a}^\infty \Phi(\zeta, \tau, \sigma) d\sigma. \tag{2.8}$$

Here

$$\Phi(\zeta, \tau, \sigma) = \frac{\exp(-\sigma\tau)}{(\sigma^2 + 4\pi^2)\zeta_0 g(\sigma)} \times$$

$$\times \frac{J_0[\zeta g(\sigma)] N_1[\zeta_0 g(\sigma)] - N_0[\zeta g(\sigma)] J_1[\zeta_0 g(\sigma)]}{J_1^2[\zeta_0 g(\sigma)] + N_1^2[\zeta_0 g(\sigma)]},$$

$$w_1 = \Delta_1 \exp\left(i\frac{\psi}{2}\right), \qquad w_2 = \Delta_2 \exp\left(-i\frac{\psi}{2}\right),$$

$$g(\sigma) = \left(\frac{\sigma(1 - a\sigma)}{1 - b\sigma}\right)^{1/2}.$$

Here J_0, J_1, N_0, and N_1 are Bessel functions. The integral terms in (2.8) may be discarded for τ large, so the steady-state pressure distribution is

$$p(r, t) = p_0(r) + \frac{Q_1 \mu}{4\pi k_1 h}\left\{\frac{K_0(X_1)\exp[i(\omega_0 t - {}^1\!/_2\pi)]}{X_{10} K_1(X_{10})} - \right.$$

$$\left. - \frac{K_0(X_2)\exp[-i(\omega_0 t + {}^1\!/_2\pi)]}{X_{20} K_1(X_{20})}\right\},$$

$$X_j = r w_j (\varkappa_1 T_0)^{-1/2}, \quad X_{j0} = r_0 w_j (\varkappa_1 T_0)^{-1/2}, \quad j = 1, 2. \tag{2.9}$$

Now $zK_1(z) \sim 1$ for $z \to 0$, so p(r, t) for X_{10} and X_{20} small becomes

$$p(r, t) = p_0(r) + \frac{Q_1 \mu}{4\pi k_1 h}\left\{K_0(X_1)\exp\left[i\left(\omega_0 t - \frac{\pi}{2}\right)\right] - \right.$$

$$\left. - K_0(X_2)\exp\left[-i\left(\omega_0 t + \frac{\pi}{2}\right)\right]\right\}. \tag{2.10}$$

For X_{10} and X_{20} we have

$$|X_{j0}| < r_0 \sqrt{2\pi(1+\delta)(\varkappa T_0)^{-1}} \qquad (j = 1, 2). \tag{2.11}$$

Here δ may vary within wide limits. The case $\delta = 0$ was considered in [1], which corresponds to negligible porosity and joint compressibility. It is found [4] that β_{c1} may be as large as $10^2 \beta_{c2}$. We assume $0 \le \beta_{c1} \le 10^2 \beta_{c2}$ and use the orders of β and β_{c2} ($\beta \approx \beta_{c2} \sim 10^{-4}$–$10^{-5}$ cm^2/sec^{-1}), to get $0 \le \delta \le 100$. Putting $r_0 \approx 10$ cm, $\varkappa \approx 10^4$ cm^2/sec, and $\delta \approx 100$, we get from (2.11) that (2.10) applies with an error of not more than 5% for $T_0 \ge 150$ sec.

3. PARAMETER DETERMINATION FROM PRESSURE CURVES

The parameters can be deduced from periodic perturbations, and the resulting pressure variations can be recorded at the wall of the source borehole ($r = r_0$) and in a test borehole ($r = R$).

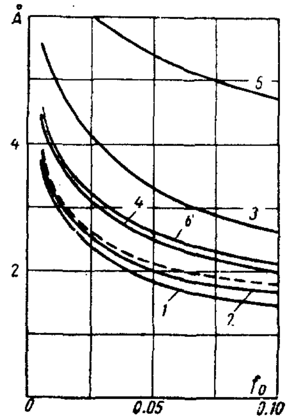

Fig. 2

We use (2.10), which contains the combination k_1h/μ, the parameter $\varkappa_1/r_0^2$ (or $\varkappa_1/R^2$), τ_0, and δ. These parameters are deduced, while μ, m_{02}, k_2, β_{c2}, and β are found from laboratory measurements; these with h and R give us k_1, $\varkappa_1$, β_1, η_1, a, r_0. These allow us [1] to deduce the mean block size l. Consider first the parameter determination from the pressure at the wall of the source borehole.

We put $r = r_0$ in (2.10) and use the above estimates to show that for the values appearing in the argument of K_0 we can put

$$K_0(z) \sim -(\ln {}^1/_2 z + 0.577\ldots) \tag{3.1}$$

for z small; from (2.10) and (3.1) we get the pressure as

$$p(r_0, t) - p_0(r_0) = A \sin(\omega_0 t - \varphi), \tag{3.2}$$

$$A = \frac{Q_1\mu}{4\pi k_1 h}\left[\psi^2 + \left(\ln\frac{1.26\varkappa_1 T_0}{r_0^2\Delta_1^2}\right)^2\right]^{1/2},$$

$$\operatorname{tg}\varphi = \psi\left(\ln\frac{1.26\varkappa_1 T_0}{r_0^2\Delta_1^2}\right)^{-1}. \tag{3.3}$$

Then for $\tau_0 = 0$ we get the expressions for A and $\tan\varphi$ given in [2] for an ordinary porous medium.

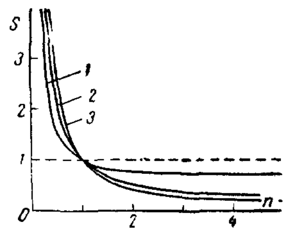

Fig. 3

Figure 2 shows $A° = (4\pi k_1 h/Q_1\mu)A$ as a function of $f_0 = r_0^2/\varkappa_1 T_0$, in which curves 1–6 correspond to various values of b and δ [1) $b = 0.1$, $\delta = 0.01$; 2) $b = 0.1$, $\delta = 1$; 3) $b = 1$, $\delta = 0.01$; 4) $b = 1$, $\delta = 1$; 5) $b = 10$, $\delta = 0.01$; 6) $b = 10$, $\delta = 1$].

The broken line corresponds to $b = 0$ (ordinary porous medium with parameters k_1 and $\varkappa_1$). For given f_0 and δ there is a nonmonotonic variation of A with b; for b small, A decreases to values less than A_0, but then A begins to increase with b and becomes larger than A_0. The difference $A - A_0$ for b large is inversely related to δ.

The observed curve may be compared with (2.1) to get A and φ.

A point of interest is to determine whether joints are present in an unknown stratum without determining the parameters. From (3.3) we readily get for an ordinary porous medium

$$S = \frac{A_1 \sin\varphi_1}{A_2 \sin\varphi_2} = 1$$

for a pair of frequencies ω_{01} and ω_{02}, for which the corresponding amplitudes and phase shifts are A_1, φ_1, A_2, and φ_2. Similarly, if the stratum is jointed

$$S = \frac{A_1 \sin\varphi_1}{A_2 \sin\varphi_2} = \operatorname{arc\,tg}\frac{1 + (2\pi b_1)^2 c}{2\pi b_1(1-c)}\left[\operatorname{arc\,tg}\frac{1 + (2\pi b_2)^2 c}{2\pi b_2(1-c)}\right]^{-1}$$

$$\left(b_1 = \frac{\tau_0}{T_{01}},\ b_2 = \frac{\tau_0}{T_{02}},\ T_{01} = \frac{2\pi}{\omega_{01}},\ T_{02} = \frac{2\pi}{\omega_{02}}\right).$$

We put $n = T_{02}/T_{01}$ and consider the function

$$S(n) = \operatorname{arc\,tg}\frac{1 + (2\pi b_1)^2 c}{2\pi b_1(1-c)}\left[\operatorname{arc\,tg}\frac{n^2 + (2\pi b_1)^2 c}{2\pi b_1(1-c)n}\right]^{-1}.$$

$S(n) \equiv 1$ for an ordinary porous medium.

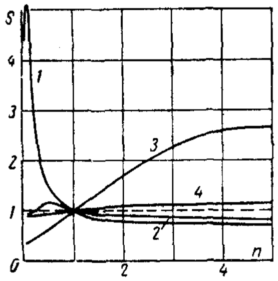

Fig. 4

Figure 3 shows S(n) for various b_1 with $\delta = 0$; curves 1, 2, and 3 correspond to b_1 of 0.1, 1, and 10. Curves 1–4 of Fig. 4 show S(n) for various b_1 with $\delta \neq 0$ [1) $b_1 = 0.1$, $\delta = 0.01$; 2) $b_1 = 0.1$, $\delta = 1$; 3) $b_1 = 10$, $\delta = 0.01$; 4) $b_1 = 10$, $\delta = 1$].

The curves show that S may differ substantially from 1 for a jointed medium of small δ, so the S(n) results for a borehole can indicate whether the rock is jointed.

As regards parameter determination, measurement of A and φ for ω_{01} and ω_{02} gives, from (3.3) a system of four transcendental equations for k_1h/μ, $\varkappa_1/r_0^2$, τ_0, δ, which may be solved by some approximate method.

The determination of $\varkappa_1/r_0^2$, τ_0, δ is much simplified if k_1h/μ is known; this may be found by the method of steady-state flow or from the asymptote of the pressure-recovery curve. If k_1h/μ is known, the A and φ for ω_{01} and ω_{02} give us from (1.17) and (3.3) a system of two linear equations:

$$\begin{aligned} \omega_{01}^2 u - \omega_{01}\,\mathrm{tg}\,\psi_1 v + 1 &= 0, \\ \omega_{02}^2 u - \omega_{02}\,\mathrm{tg}\,\psi_2 v + 1 &= 0, \end{aligned} \tag{3.4}$$

in the unknowns $u = \tau_0^2 c$, $\nu = \tau_0(1 - c)$, with

$$\psi_1 = \frac{4\pi k_1 h}{Q_1\mu} A_1 \sin\varphi_1, \qquad \psi_2 = \frac{4\pi k_1 h}{Q_1\mu} A_2 \sin\varphi_2.$$

Solution of (3.4) gives us $\tau_0, \delta, \varkappa_1/r_0^2$.

Consider now parameter determination from the pressure in a separate borehole; we here put r = R in (2.10). R is usually fairly large, and

$$\left|\frac{R}{\sqrt{\varkappa_1 T_0}} w_j\right| \gg 1 \qquad (j = 1, 2).$$

The asymptotic expression for $K_0(z)$ for z large is

$$K_0(z) \sim \sqrt{\pi/2z}\,e^{-z}. \tag{3.5}$$

Then (3.5) and (2.10) give us that

$$p(R, t) - p_0(R) = B \sin(\omega_0 t - \Theta),$$

$$B = \frac{Q_1\mu}{2\pi k_1 h}\left(\frac{\pi\sqrt{\varkappa_1 T_0}}{2R\Delta_1}\right)^{1/2} \exp\left(-\frac{R\Delta_1}{\sqrt{\varkappa_1 T_0}} \cos\frac{\psi}{2}\right),$$

$$\Theta = \frac{R\Delta_1}{\sqrt{\varkappa_1 T_0}} \sin\frac{\psi}{2} + \frac{\psi}{2}.$$

Then for ω_{01} and ω_{02} we get a system of equations for k_1h/μ, $\varkappa_1/R^2$, τ_0, δ. To conclude we note that this method demands a fairly sensitive system for recording the pressure variation.

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